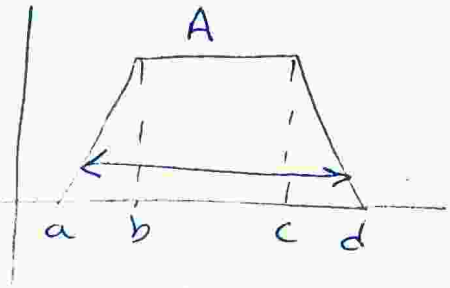
Concepts about fuzzy sets

support

elements of fuzzy set where its MF degree $\neq 0$

ex

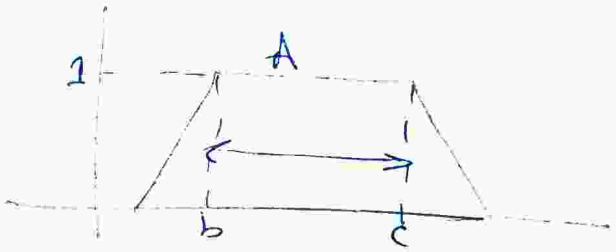
support = $]a, d[$



Core

→ elements of fuzzy set where its MF degree = 1

Core(A) = $[b, c]$



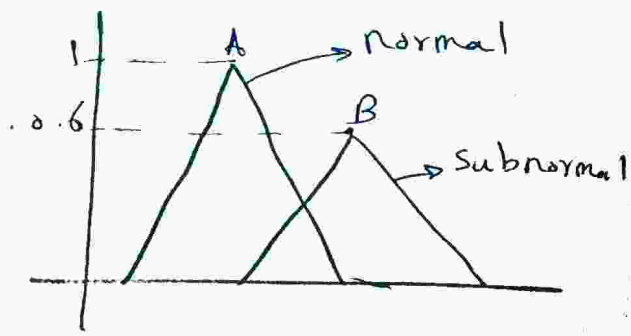
singleton

when → no. of elements of fuzzy set is equal to 1 with $\mu = 1$, it is called singleton

* Normal and Subnormal fuzzy sets

Normal → at least one element with $\mu = 1$

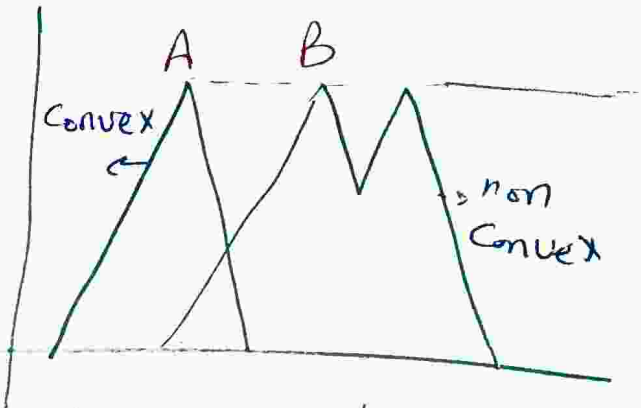
Subnormal → no element with $\mu = 1$



Convex, non Convex

Convex

~~range must be from 0 to 1~~



if μ is increase or decrease or increase and decrease over elements of set.

* we need fuzzy set to be: (in design)

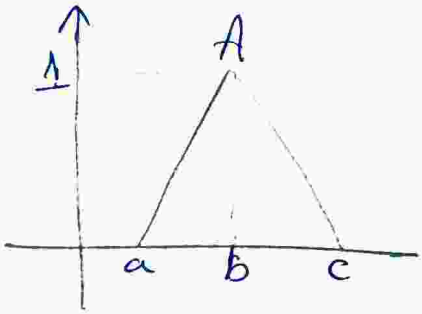
1) Nirmal. 2) Convex

3) has bounded support

Fuzzy sets

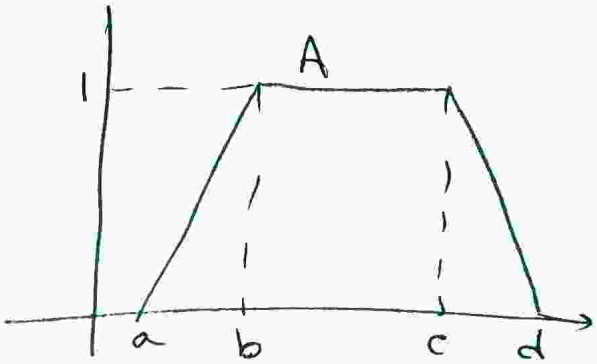
1) Triangular

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ \frac{x-c}{b-c} & b \le x \le c \\ 0 & \text{ow} \end{cases}$$



$$\mu_A(x) = \max\left[\min\left(\frac{x-a}{b-a}, \frac{x-c}{b-c}\right), 0\right]$$

2) Trapezoidal



$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \dfrac{x-d}{c-d} & c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

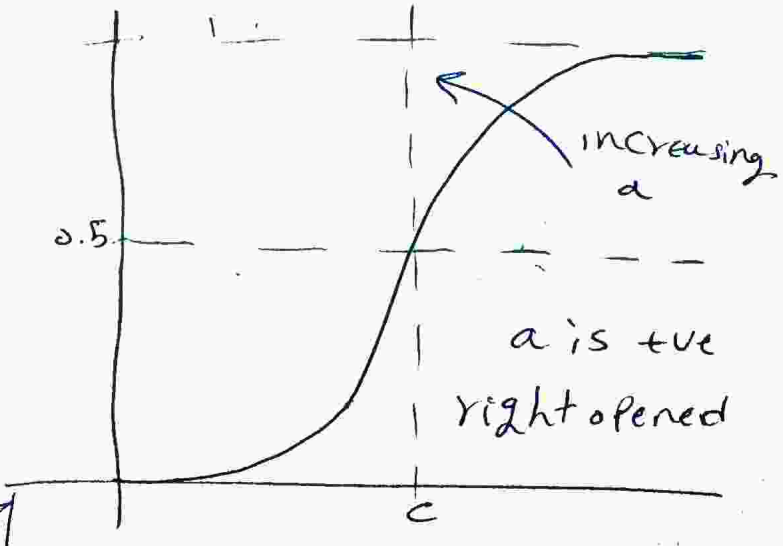
$$\mu_A(x) = \max\left[\min\left(\frac{x-a}{b-a}, 1, \frac{x-d}{c-d}\right), 0\right]$$

3) Gaussian

$$\mu_A(x) = e^{-0.5\left(\frac{x-c}{w}\right)^2}$$

4) Sigmoidal

$$\mu_A(x) = \frac{1}{1 + e^{-a(x-c)}}$$



operations

union operation

- Max operation
- Product $\mu_A + \mu_B - \mu_A \cdot \mu_B$

Intersection

- minimum $\min(\mu_A, \mu_B)$
- ~~Max~~ Product $\mu_A \cdot \mu_B$

Complement

$$\mu_{\bar{A}} = 1 - \mu_A$$

Fuzzy

slides نظري از

Advantages of fuzzy controllers:-

1) cheap in cost.
2) more robustness
3) Customizable.
4) easy to design & implement

Fuzzy APPs

* washing machine.
* Rice Cookers.
* microwave ovens.
* Vacuum cleaners.

} Consumer Product

* Elevators
* Cranes
* train
* traffic control.

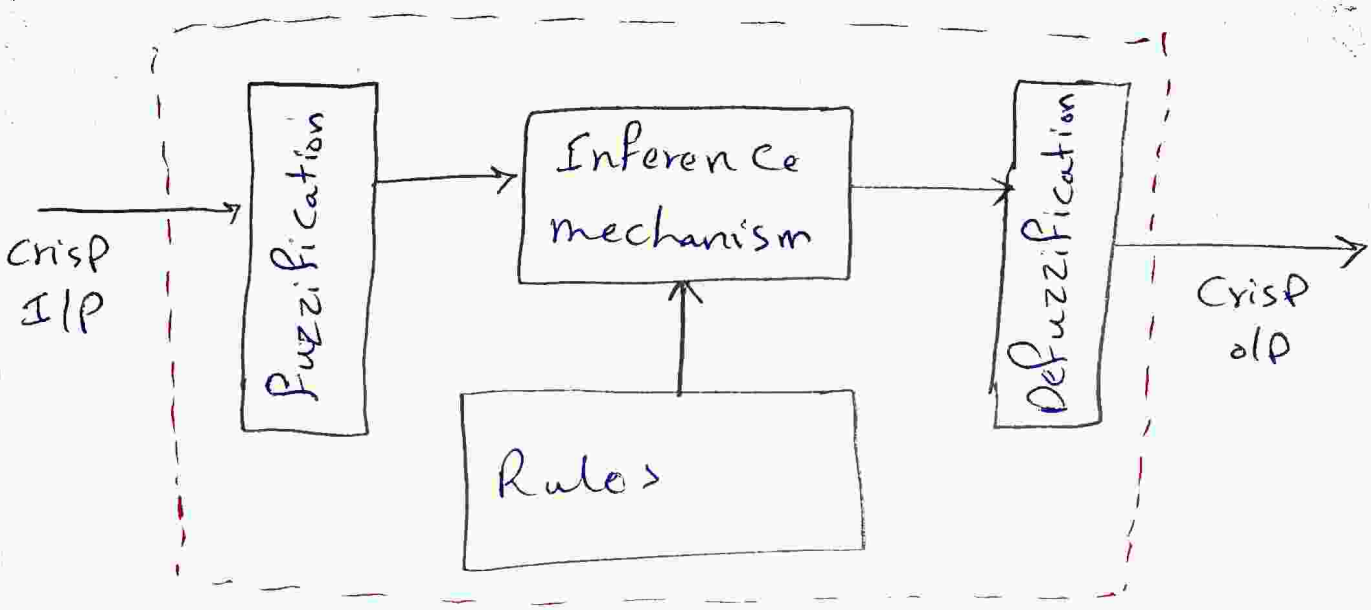
} → systems

* Difference between fuzzy & classical (binary) sets

→ binary set: every element is member or a non-member of the set.

→ fuzzy set: every element can be member of some degree and at the same time non-member to some degree at the same set.

$$\mu_A(x) \in [0, 1]$$



Fuzzy Controller

1] Fuzzification

→ Process of converting crisp values of fuzzy controller inputs into a fuzzy input sets.

~~→ It's output is~~

2] Rules & inference mechanism

→ Rules : set of if-then statements including expert's linguistic description that governs the performance of controller.

→ Inference mechanism (heart of fuzzy control) emulates the expert's decision making in interpreting and applying knowledge about best to control plant.

→ ~~Itvalso~~

(3) Defuzzification

→ inverse process of fuzzification (convert fuzzy quantity into crisp value)

Rules

if ⇒ part ⇒ contains no. of existed potentials

then part ⇒ " the actions

* Design steps of fuzzy controller (Fc)

1) Determine inputs and outputs of fuzzy controller. and desired input of system.

2) choose shapes and universe of discourse of fuzzy sets for inputs and output of Fc.

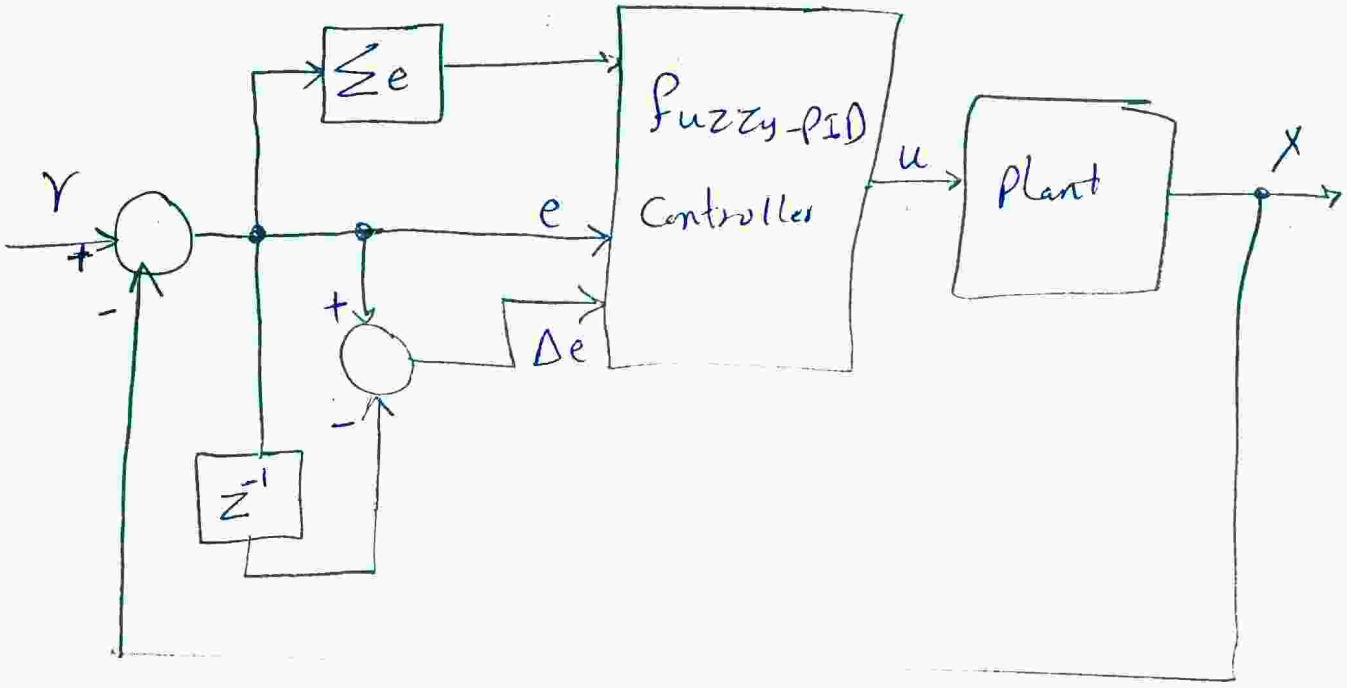
3) write suitable rules.

Fuzzy ~~PID~~ PID Controller

→ used to obtain better performance.

→ basic idea of ~~PID~~ it to chose control law by considering error e, change of error De and integral of error

$$u_{PID} = K_p \cdot e + K_D \cdot \Delta e + K_I \cdot \int_0^t e \cdot dt$$



*chosing shapes of fuzzy sets I/p & o/p for fuzzy controller :-

1) use normal fuzzy sets.

2) use symmetrical triangular fuzzy sets with 50 % overlap.

3) It is prefer to choose odd number of fuzzy sets (3, 5, 7, ...)

4) To ensure that universe of discourse covers all possible values of i/ps for fuzzy controller.

Inference: Process of formulating a nonlinear mapping from given crisp i/p to crisp output.

Types of Fuzzy inference

1) Mamdani Fuzzy inference } most commonly
2) Sugeno " "
3) Tsukamoto " "

← التعامل يكون لا نهائي (nonlinear) لأنه يستخدم العمليات الحسابية
كلما تكون (linear).

1 Mamdani

→ ~~rules~~ rules obtained from an experienced human operator

ex

R_1: if X_1 is A_1 and X_2 is B_1 then y is C_1

output of each rule is the truncated membership functions from minimum firing strength.

methods of defuzzification (in Mamdani)

a) center of gravity (CoG)

b) weighted average method

c) Mean-max membership method.

2] Sugeno or TSK fuzzy inference:

rules linear function which is combination of input variables plus constant term.

EX

R1: if x_1 is A_1 And x_2 is B_1 then $Y_1 = p_1 x_1 + q_1 x_2 + r_1$

defuzzification (weighted average)

3] Tsukamoto fuzzy inference

rules

↳ It is monotic MF

ex

R_1: if x_1 is A_1 and x_2 is B_1 then y is C_1

defuzzification

weighted average.

main difference between 3 ways

1) Rules (then Part)
and write rules of each way.
2) way of apply defuzzification.

---

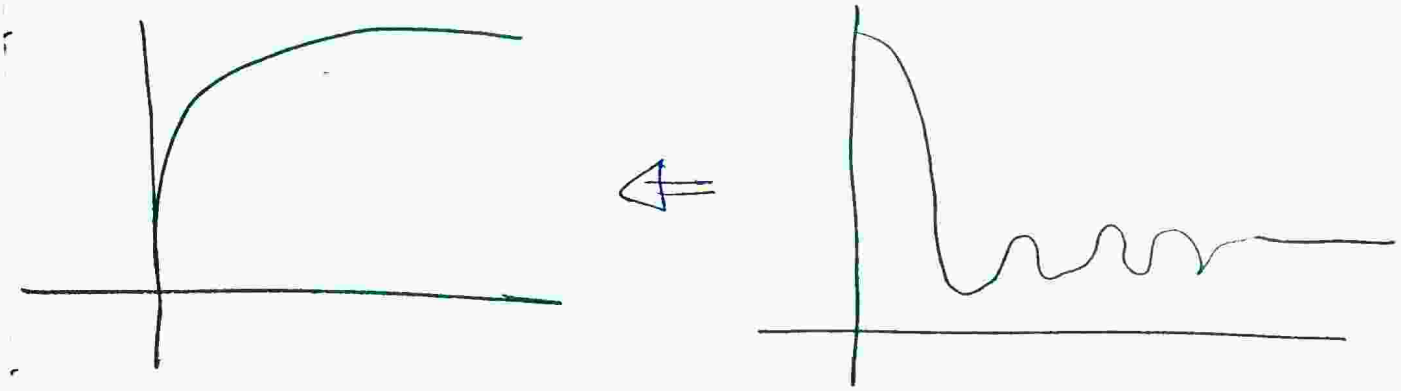
* لو ال (o/p) بسبب ال (fuzzy) يروح لل (steady state) بس بالسالب فيه بحلول:-

1) تغير ال (gain) للسالب.

2) تعكس ال (rules) ⇐ عند جمع ال (rules) مش محط عكس المجموع الجبري.

3) اعكس مفهوم ال (error)

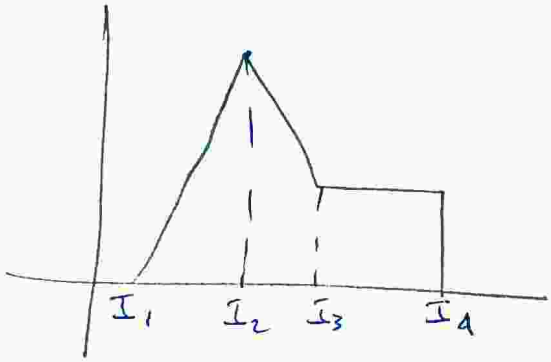
$e = r - y \Rightarrow e = y - r$



ways of defuzzifications.

Center of gravity

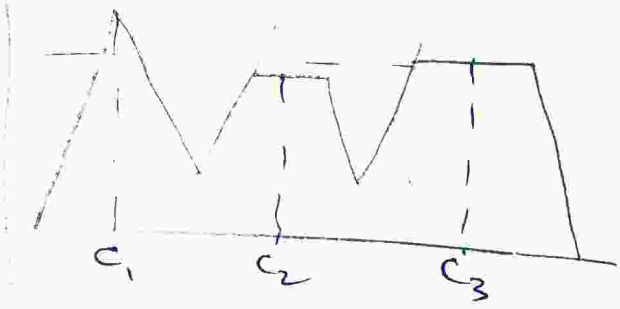
$$u^{crisp} = \frac{\int \mu(u)\, u\, du}{\int \mu(u)\, du}$$



← كل فترة وتكاملها

~~Mean~~ Max - mean

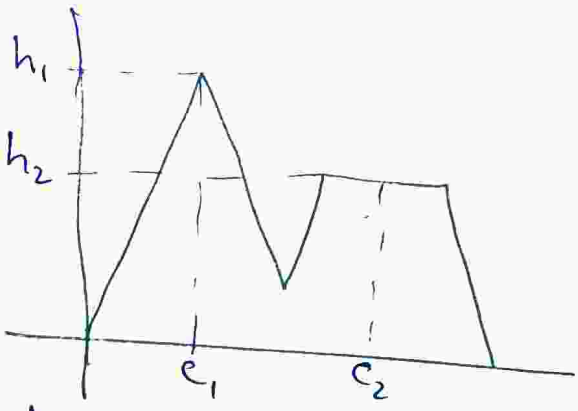
$$u^{crisp} = \frac{c_1 + c_2 + c_3}{3}$$



maximum لا القيم الكبيرة

weighted average

$$u^{crisp} = \frac{c_1 h_1 + c_2 h_2}{h_1 + h_2}$$



لو فيه قائل تستخدمها.

Fuzzy-PID Controller

↳ is used to obtain better performance in respect of rise time, settling time, overshoot and steady-state error.

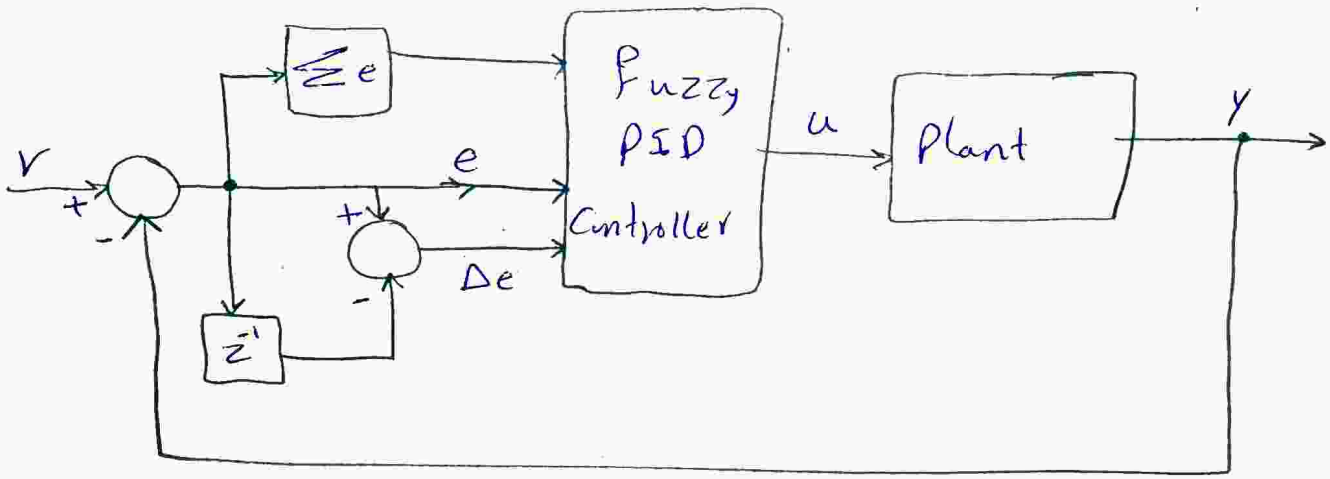
Basic idea of PID

↳ to choose control law by considering error, change of error and integral of error.

$$u_{PID} = K_p \cdot e + K_D \cdot \Delta e + K_i \int_0^t e\,dt$$

for discrete:

$$u_{PID} = K_p \cdot e + K_D \cdot \Delta e + K_i \sum e$$



← الـ (system) ده مش مناسب من الناحية العملية لأن عندك (huge no. of rules) يعني (3 inputs) يعني

(fast response) في حين ان التطبيقات محتاجة (long reasoning time)

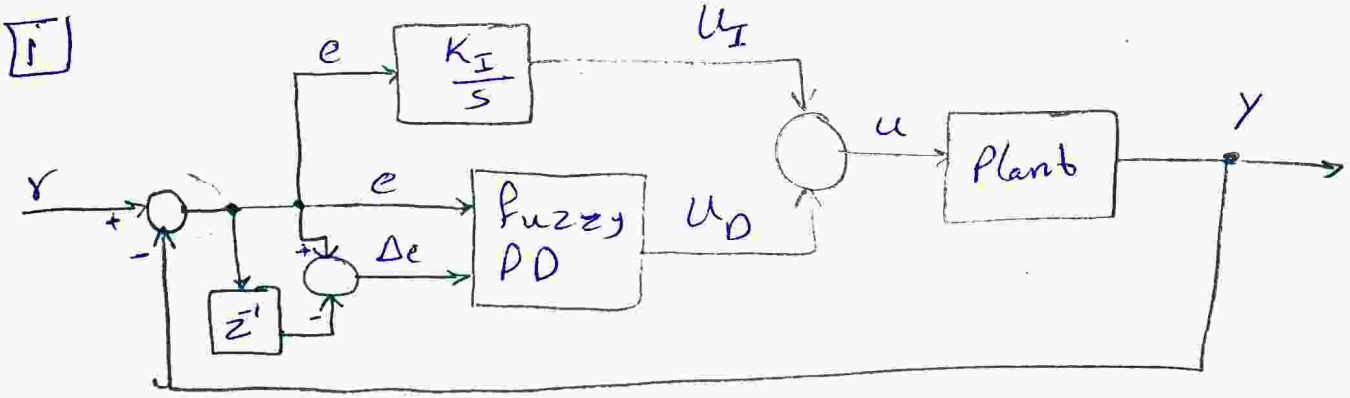
→ There are various methods used to design fuzzy-PID Controller, but the two most common methods:

1) Combination fuzzy-PD Controller with Conventional integral Controller.

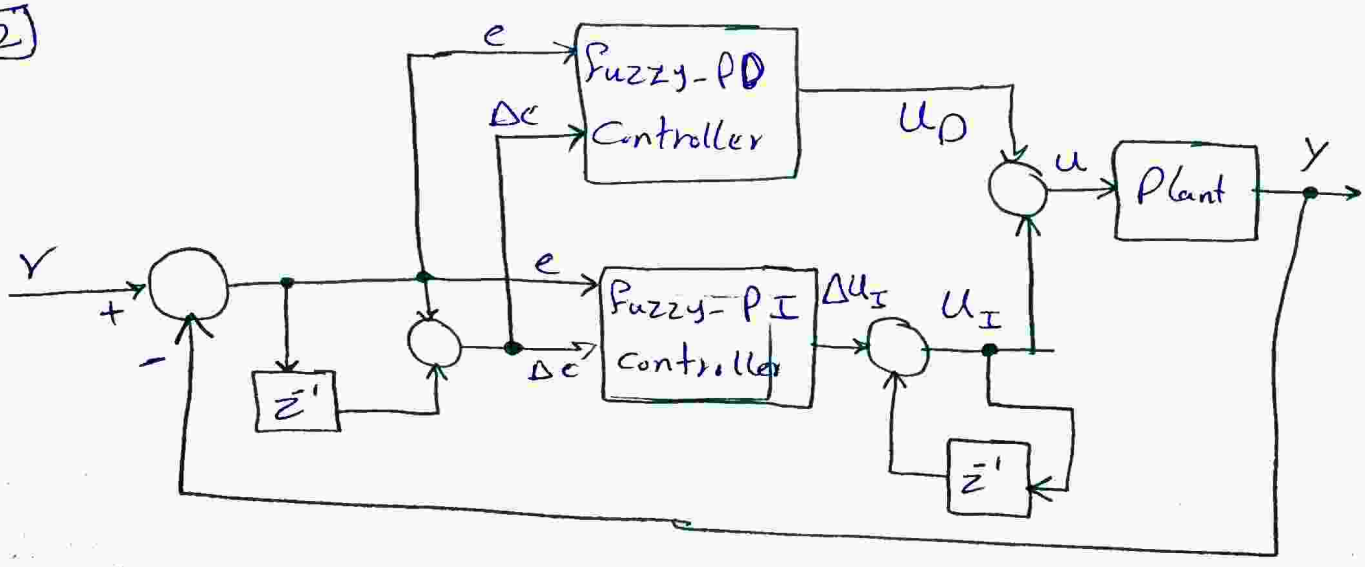
2) Combination of fuzzy-PD Controller with fuzzy-PI Controller.

⇒ In Conventional integral Controller (gain K_i is to be determined by trial and error)

1



2



ANFIS

Evolutionary Computation

↳ Heuristic algorithms based on Principles of Darwinian evolution

Example of these algorithms :-

a) Genetic algorithm. b) Differential evolution algorithm.

c) Fish swarm. d) Artifial Bee Colony.

e) Particle swarm.

ANFIS based on TSK fuzzy inference system

↳ AnFIS is fuzzy system modelled in the form of the artifial neural network.

⇒ Why ANFIS is based on TSK fuzzy inference?

Cause ~~Fuzzy~~ TSK is simple in Computation and easy to be Combined with optimizing and self-adapting methods.

ANFIS stands for :

↳ Adaptive neuro fuzzy inference system.

↳ Adaptive network-based fuzzy inference system.

* What is the main objective of ANFIS?

↳ It is to determine the optimum values of the equivalent fuzzy inference system parameters (of TSK type) by applying a learning algorithm using input-output datasets (train and test)

* what are the parameters to be obtimized in ANFIS?

1) Premise (if part) which describe shape of MFs.

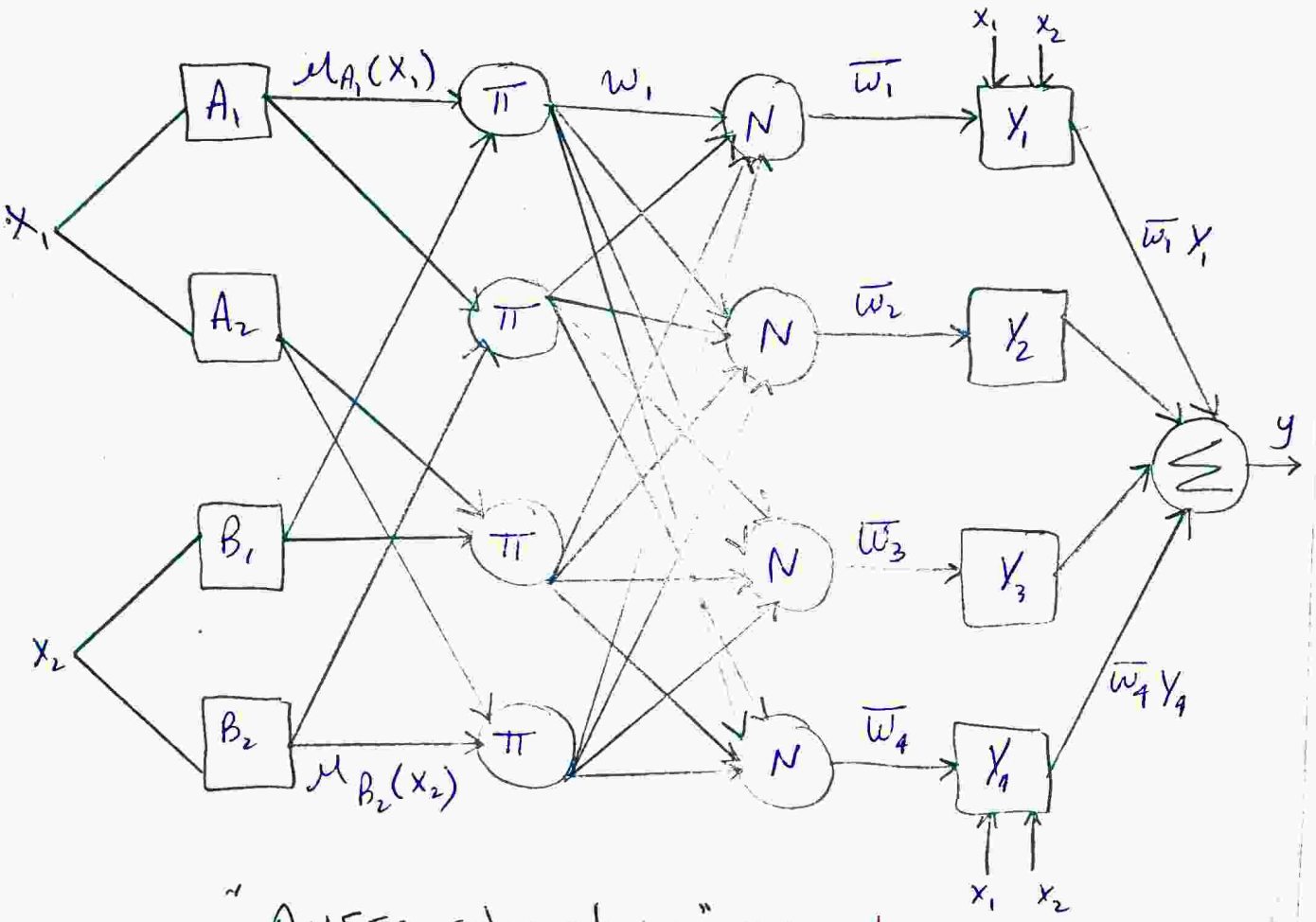
2) Consequent (then part) " " overall output of system.

* Explain with example the type of methods that used to optimize ANFIS parameters?

1) derivative-based methods
   - ↳ back propagation (BP)
   - ↳ least squares estimate (LSE)
   - ↳ hybrid learning (HL) combination of LSE & BP

2) derivative-free methods
   - ↳ genetic algorithm (GA)
   - ↳ Particle swarm optimization (PSO)
   - ↳ differential evolution (DE)
   - ↳ shuffled frog leaping Algorithm (SFLA)
   - ↳ artifial bee colony algorithm (ABC)

input layer | fuzzification layer | Rule layer | Normalization layer | defuzzification | Sum layer



"ANFIS structure" 2 inputs

Layer 0 : input layer

↳ as inputs are applying to system.

Layer 1 : fuzzification layer

↳ applying inputs MFs and Produce a degree of membership. (μ)

Layer (2) : Rule layer

↳ its output represents fire strength of rule.

↳ executes fuzzy of antecedent (if) part

Layer 3: normalization layer

↳ output is the ratio of firing strength of ith rule to sum of all firing strengths rule.

$$\bar{w}_i = \frac{w_i}{w_1 + w_2 + w_3 + w_4} \quad , i = 1, 2, 3, 4$$

Layer 4: defuzzification layer:-

↳ it executes consequent part of fuzzy rules

↳ its output is product of normalized firing strength rule & its corresponding linear function in consequent part.

Layer 5: Sum layer:-

↳ computes total crisp output of fuzzy system.

$$y^{crisp} = \frac{w_1 Y_1 + w_2 Y_2 + w_3 Y_3 + w_4 Y_4}{w_1 + w_2 + w_3 + w_4} = \bar{w}_1 Y_1 + \bar{w}_2 Y_2 + \bar{w}_3 Y_3 + \bar{w}_4 Y_4$$

Notes

* no. of premise parameters = no. of control parameters of MFs * total no. of MFs
* no. of consequent parameters = (no. of inputs + 1) * no. of rules
* total no. of ANFIS parameters = no. of premise parameters + no. of consequent parameters.